Real and Imaginary Numbers

In equation-based mathematics a distinction is drawn between two kinds of mathematical object, real (±1) and imaginary (±√-1. The two oppositional (+ and -) forms of each kind mutually negate to a third mathematical object, nothing (0), which transcends the distinction drawn between the other two kinds of mathematical object.

The three kinds of mathematical object constitute an artefact of working on the principle of a balance. Properly there are three oppositional pairs of relational mathematical object constituted in the symmetry of Order 4.

All of the kinds of mathematical object are outputs from four input pairs (01 or -+, 10 or +-, 11 or ++ and 00 or --). Distinctions are drawn using the truth tables for XOR and its mirror opposite XNOR.

	XOR			
	0 means same			
	1 means distinct			
Input 1	1	0	1	0
Input 2	0	1	1	0
Output	1	1	0	0

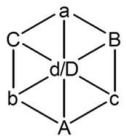
	XNOR			
	1 means same			
	0 means distinct			
Input 1	0	1	0	1
Input 2	1	0	0	1
Output	0	0	1	1

	XOR			
±1	Real			
	- means distinct			
Input 1	-	+	-	+
Input 2	+	-	-	+
Output	-	-	+	+

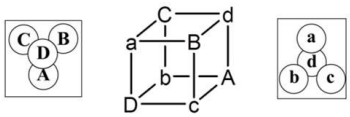
	XNOR			
±√1	Imaginary			
	+ means distinct			
Input 1	+	-	+	-
Input 2	-	+	+	-
Output	+	+	-	-

Construction of Zero, the Real Numbers and the Imaginary Numbers from Four Input Pairs on the Principle of Balance

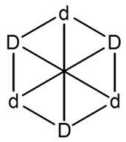
There are six possible ways of ordering four objects, and hence the four input pairs, in rings (Order 4). This forces the truth table for XOR and XNOR to interpenetrate, such that the meanings of 1 and 0 oscillate relationally.



ORDER 4
One distinction (that between d and D) is lost when four different objects are constrained to be ordered in rings.



ORDER 4
Two interpenetrating enantiomers of a chiral tetrahedron:
(a, b, c and d) and (A, B, C, D).



ORDER 4
The lost distinction between d and D can only be recovered at the expense of losing the distinctions defining a, A, a; B, b and c, C.

Interpenetrating XOR and XNOR Truth Tables
Oscillation of meanings of 1 and 0

Triplet state	0	1	0	1
	1	1	0	0
	1	0	0	1
	1	0	1	0
	0	0	1	1
	0	1	1	0

Construction of Real Numbers, Imaginary Numbers and Zero as relational Objects in Order 4

Interpenetrating XOR and XNOR Truth Tables
Oscillation of meanings of 1 and 0

Singlet state	0	1	0	1
	1	0	1	0
	0	1	0	1
	1	0	1	0
	0	1	0	1
	1	0	1	0

This relational reconstruction redraws the distinctions defining zero, real and imaginary mathematical objects to create three symmetrical oppositional pairs of relational mathematical objects that behave like an oppositional pair of zeroes (a, A), an oppositional pair of real numbers (B, b) and an oppositional pair of imaginary numbers (c, C). In effect it integrates zero, real and imaginary numbers into a relational cycle, for example from (a, A) to (A, a), analogous to the creation and redemption of money and debt from balance (zero) in a cycle of exchange of goods and services.

$$(a,A) \rightarrow (B, b) \rightarrow (c, C) \rightarrow (A,a)$$

$$(0) \quad \rightarrow \quad (+1, -1) \quad \rightarrow \quad (0)$$