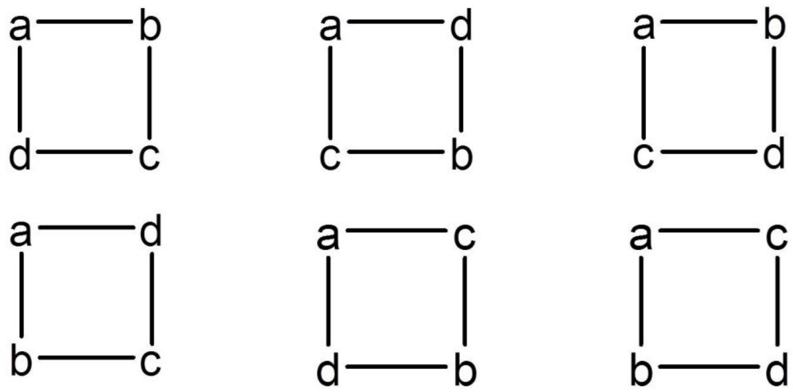
Order 4

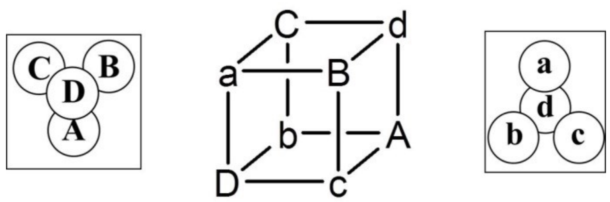
Four different objects - a, b, c, d - can be ordered 24 (4 x 3 x 2 x 1) ways in a chain, reading in one direction, but only 6 (3 x 2 x 1) ways in a ring. A cube has six faces. If the corners of each face are marked with a, b, c and d, then each face will be one of these six rings. Rotation of a face conserves the order of the letters. Rotation of the cube from one face to another changes the order of the letters from one ring to another.

The six rings form into three pairs, one in which the order of the four letters is read clockwise and one in which the order is read anticlockwise.



ORDER 4
Six ways of ordering four different objects
(a, b, c and d) clockwise or anticlockwise in a ring.

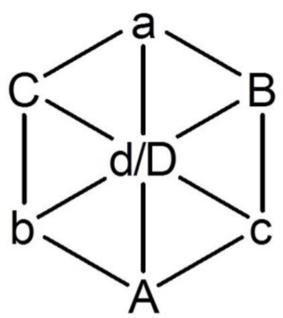
Instead of pairing the rings by drawing a distinction between a clockwise and anticlockwise sense of rotation, rings can be paired by drawing a distinction based on mirror symmetry, which separates front and back, closing the distinction drawn between a clockwise and anticlockwise sense of rotation. To do this, we note that the corners of a cube define two interpenetrating enantiomers (left- and right-handed forms) of a tetrahedron. We can label the corners of these tetrahedrons a, b, c and d and A, B, C and D.



ORDER 4
Two interpenetrating enantiomers of a chiral tetrahedron:
(a, b, c and d) and (A, B, C, D).

In the tetrahedron a, b, c and d, when d points to the back a, b and c read anticlockwise. In the other tetrahedron A, B, C and D, when D points to the front A, B and C also read anticlockwise. The distinction between clockwise and anticlockwise sense, which draws no distinction between the two a, two b, two c and two d has been replaced by distinctions between them: a, A, b, B, c, C and d, D. The pairings a, A; b, B; c, C and d, D each define two sides of one object - a relationship. Each face consists of two lower case and two upper case letters in alternation and its opposing face has the opposite arrangement rotated through 180 degrees, for example a, B, c, D and A, b, C, d, rotated through 180 degrees.

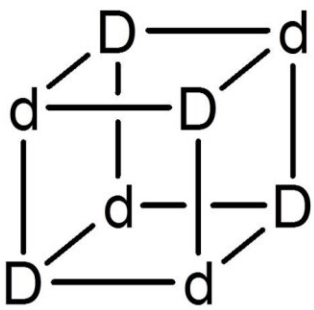
Now imagine rotating the cube so that d and D are aligned one over the other. In projection the other corners of the cube now align with the six corners of a hexagon, so forming a ring. Each corner is now mapped to each of the six possible orders of the four different objects in a ring. This has come at the cost of losing the distinction between d and D.

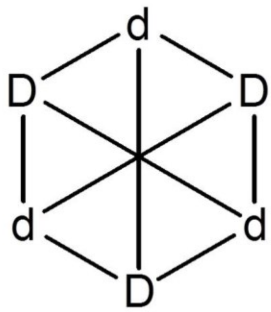


ORDER 4
One distinction (that between d and D) is lost when four different objects are constrained to be ordered in rings.

Next imagine that the corners of the two interpenetrating tetrahedrons in the cube are the same – d, d, d and d and D, D, D and D. The tetrahedrons cannot be distinguished by the sense of rotation: clockwise and anticlockwise. The only distinction drawn is that between d and D. Drawing this distinction between d and D comes at the price of losing the distinctions between a, A; b, B and c. C.

The symmetry constraint imposed by Order 4 - the ordering of four distinct objects in a ring – creates two relational objects: one in which three relational objects are defined (a, A; b, B and c. C) and one in which one relational object is defined (d, D).





ORDER 4
The lost distinction between d and D can only be recovered at the expense of losing the distinctions defining a, A; B, b and c, C.

The relational symmetry inherent in Order 4 requires both of the relational objects to define it: one constituted in 3 relational pairings and one constituted in 1 relational pairing - 4 altogether - to define it.

Summary

An object (x) is defined by a relationship formed by drawing a distinction between what it is (x) and what it is not (X). This relationship can be reformed by erasing the distinction (x,X) and drawing a new distinction (y,Y). Four distinct objects, a, b, c and d are each defined by a relationship between what each is (x) and is not (X): (a,A, b,B, c,C and d,D). Symmetry constraints imposed by the six possible ways in which four different objects can be reordered restricts what can happen when distinctions are reformed. The system will always transform such that there are three pairs of relational objects in a ring (a, B, c, A, b, c) and one other relational object (d, D) that separates the two sides of the relational objects (a from A, b from B and c from C). This is universal.